Distributive pluractionality

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What is distributivity?

Some distributive vocabulary

- **Key:** The thing we break up into parts.
- **Share:** The thing the parts individually participate in.
- **Map:** The relation between key and share

(1) The students each left.
   a. **Key:** the students
   b. **Share:** left
   c. **Map:** agent theta-role

(2) two pounds of tomatos.
   a. **Key:** two pounds
   b. **Share:** tomatos
   c. **Map:** weight

Some distributive vocabulary

I will use "distributive operator" very generally for expressions whose interpretation requires a key-share-map relation.

(3) a. The students left **one by one**.

b. The students **each** left.

c. **Every **student left.**

d. **The students left** individually**.

An aside on the D operator

Some verbs, so called "mixed predicates", allow both distributive and collective readings.

(4) The children lifted the table
    a. **True** in the teamwork scenario
    b. **True** in the test-of-strength scenario

For account for the latter, many have assumed there is a covert distributivity operator.

(5) The children **D** lifted the table

I okay with this as a kind of descriptive way to mark that there the predication at hand is a distributive sort, but I am not ready to claim there is actually an operator here.

Kinds of distributive operators

I group distributive operators into four rough categories:

- distributive quantifiers: each, every, most, etc.
- distributive modifiers: one by one, individually, "floated each" etc.
- distributive predication: e.g., the distributive interpretation of mixed predicates with plural nominals.
- distributive pluractionals: **to be discussed**

Kinds of distributive operators

Within these kinds there can also be important distinctions. First, I want to distinguish distributive operators that are key-marking and those that are share-marking.

(6) a. **Each** boy ate an apple. <− key-marking
    b. The boys **each** ate an apple. <− share-marking
    c. The boys ate an apple **each** <− share-marking

Note!! This distinction does not change what are the key and shares.

Kinds of distributive operators

It is also important to distinguish those operators that merely distribute the share over the (atomic) parts of the key and those that also impose other constraints.

(7) The students **each** danced.
Each student gets their own leaving event.

(8) The students danced **one by one**.
Not only does each student get their own leaving event, but those events must be ordered in time.

Thinking about pluractionals

When we turn to pluractionals, classification into these subtypes will become interesting.

- Because pluractionals, by definition, are verbal morphology, they must either be share-marking or must be operating over the verb—i.e., the opposite of quantifier *each*.
- It would also be interesting to see whether pluractionals induce additional entailments about the share or key. I believe this to be commonly the case.

Distributive Strength

Distributive Strength

We can rank distributivity (operators) by strength. The stronger the distributivity, the more strictly it controls its share—consider the following pairs of examples:

(9) a. All the students gathered in the park.

  b. #All the students are numerous.

  c. #John gathered in the park.

(10) a. #Each of the students gathered in the park.

  b. #Each of the students are numerous.

  c. # John is numerous.

The predicates *gather* and *be numerous* cannot have atomic individual participants. The distributive operator *each of* forces the share to to be predicated of atoms in the key in a stricter way than *all* does.

Distributive Strength

Taking this lesson to heart we can sort distributive operators by strength.

- We have: **each** > **all**

Is there any reason to think that these quantificational distributive operators are different than predicative distributivity?

(11) Suppose every single student ordered their own pizza to eat.

    a. Each student ate a pizza.

    b. All the students ate a pizza.

    c. #The students ate a pizza.

We see here that predicative distributivity cannot "scope over" an indefinite. Another way to say this is that it isn't strong enough to trap an indefinite in the share. We thus have:

- **each** > **all** > **pred.dist**

Distributive Strength

What can we say about distributive modifiers? They pattern with predicative distributivity on many tests.

(12)

    a. The students gathered **one by one**.

    b. The students ate a pizza **one by one**.

But they seem (maybe) stronger than this.

(13)

    a. The students elected the president.

    b. #The students elected the president one by one.

    c. #All the students elected the president.

    d. #Each of the students elected the president.

Distributive Strength

We thus have a strength ranking, where (roughly) quantificational distributivity is strong than adverbial distributivity is stronger than predicational distributivity.

- **each > all > n-by-n > pred.dist**

I expect that, just as we see quantificational distributors varying in strenth, I bet we see something similar for adverbials, but I believe in this general ranking.

- **quantificational > modificational > predicational**

Some questions for pluractionals

We can now ask a bunch of questions:

- How strong are pluractional distributivity operators? Are they like quantifiers, adverbial modifiers, or vanilla predicative distributivity?
- Do pluractional distributivity operators share-markers or key-markers?
- Do pluractionals only encode distributivity, or do they also impose additional constraints on the share (or key)?

A case-study of pluractional distributivity

A case-study of pluractional distributivity

Consider the following minimal pair in Kaqchikel.

(14) a. X-e'-in-q'etej ri ak'wal-a'. CP-A3p-E1s-hug the child-PL I hugged the
          children.

      b. X-e'-in-q'ete-la' ri ak'wal-a'. CP-A3p-E1s-hug-PDIST the child-PL I
          hugged the children individually.

The first is true in both situations where:

▸ I hug the children as a group

▸ I hug each children individually

The second is is only true in the later situation. It is false if I, in fact, gave the
children a group hug.

A case-study of pluractional distributivity

At an appropriately coarse level of semantic granularity, **??** is comparable to the following English sentences, where distributivity is enforced through various means.

(15) a. I hugged each child.

   b. I hugged the child one by one.

   c. I hugged the child individually.

   d. etc.

A case-study of pluractional distributivity

While English has a variety of ways to force distributive predication, the Kaqchikel example presents a route to distributivity that is absent in English, namely **pluractionality**.

(16) X-e'-in-q'ete-la' ri ak'wal-a'.
      CP-A3p-E1s-hug-PDIST the child-PL
      I hugged the children individually.

We know the -la' marker is, in fact, pluractional, because it derives verbs that cannot be satsified in single-event scenarios.

(17) X-e'-in-q'ete-la' ri ak'wal-a'.
      CP-A3p-E1s-hug-PDIST the child-PL
      I hugged the children individually.

A first-pass look at this sentence again makes it seem like:

 ▸ ri ak'wala' *the children* is the distributive key
 ▸ q'ete *hug* is the distributive share
 ▸ the theme theta-role is the map

Thus, the pluractional looks like a kind of distributivity operator that marks the share.

A case-study of pluractional distributivity

I will argue that this is not the case!

- ▶ Pluractionality will require that the event-predicate be the key, while the nominal is the share
- ▶ it is in fact the opposite of how we usual think of distributive constructions

A case-study of pluractional distributivity

Generalization 1: A plural object can interact with the distributive pluractional, but a plural subject can't.

A case-study of pluractional distributivity

Here we see that a plural object can interact with the distributive pluractional.

(18) X-e'-in-tun-ula' ri q'ul.
CP-A3p-E1s-fold-PDIST the blanket
I folded the blankets individually.
False if I folded any subset of the blankets simultaneously

(19) X-e'-in-kan-ala' ri wuj.
CP-A3p-E1s-search-PDIST the book
I searched for the books individually.
False if I looked for any subset of the books simultaneously

(20) X-e'-in-kam-ala' ri sanik.
CP-A3p-E1s-kill-PDIST the ant
I killed the ants individually.
False if I killed any subset of the ants simultaneously

A case-study of pluractional distributivity

Here we see that a plural subject can't interact with the distributive pluractional.

(21) X-∅-qa-kan-ala' ri wuj.
CP-A3s-E1p-search-PDIST the book
#We searched for the book individually.

(22) X-∅-qa-tun-ula' ri q'ul.
CP-A3s-E1p-fold-PDIST the blanket
#We each folded the blanket.

A case-study of pluractional distributivity

Generalization 2: Distributive pluractionals cannot create derived plurals (i.e., their distributivity is weak)

A case-study of pluractional distributivity

There are multiple ways of being plural. A nominal can be plural in virtue of its morphology or semantic class (i.e., plural morphology, group nouns, etc.). Alternatively, plurals can be derived by taking narrow scope, like the indefinite below.

(23) Every five minutes I ate a tortilla. My sister made them/*it for me.

A case-study of pluractional distributivity

Crucially, pluractional distributivity cannot create derived plurals. It cannot scope over the indefinite object.

(24) X-in-kan-ala' jun wuj.
CP-E1s-search-PDIST a book
I searched for a book in various places.
For example, if I spent all afternoon looking all over the house for a particular book.

(25) X-in-tik-ila' jun che'.
CP-E1s-plant-PDIST a tree
I planted a tree various places.
For example, if the boss kept telling me to move the tree somewhere else after every time I planted it.

A case-study of pluractional distributivity

The effect is even clearer with predicates of destruction, which are infelicitous with
pluractional distributive derivational morphology and singular objects.

(26) #X-in-kam-ala' jun sanik.
CP-E1s-kill-PDIST a ant
I killed an ant various times.

(27) #X-in-qum-ula' jun mama "ak'.
CP-E1s-drink-PDIST a beer
I drank a beer various times.

A case-study of pluractional distributivity

These examples we have just seen (and partially repeated below) show Generalization 3.

Generalization 3 (to be amended): Distributive pluractionals with atomic objects are not ungrammatical, but have repetition readings.

(28) X-in-tik-ila' jun che'.
CP-E1s-plant-PDIST a tree
I planted a tree various places.

A case-study of pluractional distributivity

Given that the pluractional cannot take scope over an indefinite, we can show quite easily that a plurality of events is indeed required.

Generalization 4: Distributive pluractionality requires a plurality of events.

(29) Suppose I plant a tree just once and then walk away.
      #X-in-tik-ila' jun che'.
      CP-E1s-plant-PDIST a tree

I planted a tree various places.

A case-study of pluractional distributivity

Summary of the generalizations

1. Obligatory distribution over plural objects
2. Repetition with singular objects
3. Cannot create derived plurals (narrowest scope)
4. Plural events

Case Study: Analysis

Case Study: Analysis

I work with classical many-sorted type logic with events and theta-role functions.

- the domain of individuals of type e is the powerset of a designated set of entities IN minus the empty set: $D_e = \wp^+(\mathrm{IN}) = \wp(\mathrm{IN}) \setminus \varnothing$

- the domain of events of type ϵ is the powerset of a designated set of events EV minus the empty set: $D_\epsilon = \wp^+(\mathrm{EV}) = \wp(\mathrm{EV}) \setminus \varnothing$

- atomic individuals and atomic events are the singleton sets in $\wp^+(\mathrm{IN})$ and $\wp^+(\mathrm{EV})$ respectively; they are identified by a predicate **atom** (which applies to both individuals and events)

Case Study: Analysis

- the "part of" relation $\leq$ over individuals / events is set inclusion over $\wp^+(\mathrm{IN})$ / $\wp^+(\mathrm{EV})$: $a \leq b$ iff $a \subseteq b$

- the sum operation $\oplus$ is set union over $\wp^+(\mathrm{IN})$ / $\wp^+(\mathrm{EV})$: $a \oplus b := a \cup b$

- θ-roles are functions of type ϵe from events (type ϵ) to individuals (type e)

Case Study: Analysis

I assume that arguments and adjuncts are event modifiers:

- they have denotations of type $(\epsilon t)(\epsilon t)$
- they have translations of the form $\lambda \mathcal{P}_{\epsilon t}.\lambda \mathbf{e}_\epsilon.$
$P(e) \wedge \ldots$

Further, verbs denote cumulated predicates of events:

- they have denotations of type (ϵt)
- they have translations of the form $\lambda \mathbf{e}_\epsilon . \mathcal{P}(\mathbf{e})$

Case Study: Analysis

As derivational morphology, pluractional distributivity applies to predicates of events and encapsulates a θ-role function through which it encodes distributive dependencies.

(30) PDIST $\leadsto \lambda \mathcal{P}_{\epsilon t}.\lambda \mathbf{e}_\epsilon.$
$P(e) \wedge$
$|\{e' \leq e : \mathbf{atom}(e')\}| > n \wedge$
$\forall e' \leq e(\mathbf{atom}(e') \rightarrow \mathbf{atom}(\mathbf{th}(e')))$

The contribution of PDIST is given by the final two conjuncts of 36

▸ $|\{e' \leq e : \mathbf{atom}(e')\}| > n$ is the pluractionality requirement — the predicate will only be true of events whose atomic subparts are more numerous than some contextually specified standard.

▸ $\forall e' \leq e(\mathbf{atom}(e') \rightarrow \mathbf{atom}(\mathbf{th}(e')))$ establishes a θ-based correspondence between atomic subparts of the event and atomic subparts of the range of the **th**-role.

An example

The following example has a indefinite plural object and a derived PDIST predicate.

(31) X-e'-in-q'ete-la' oxi' ak'wal-a'.
CP-A3p-E1s-hug-PDIST three child-PL
I hugged three children individually (many times).

(32) $q'ete \rightsquigarrow \lambda\mathbf{e}_\epsilon.\text{HUG}(\mathbf{e})$

(33) $oxi'^{\mathbf{th}} \rightsquigarrow \lambda X_{et}\lambda\mathcal{P}_{\epsilon t}\lambda\mathbf{e}_\epsilon.\mathcal{P}(\mathbf{e})\wedge$
$\exists x_e(|\{x' \leq x : \mathbf{atom}(x')\}| = 3 \wedge X(x) \wedge \mathbf{th}(\mathbf{e}) = x)$

(34) $ak'wala' \rightsquigarrow \lambda x_e.\text{CHILD}(x)$

An example

The following example has a indefinite plural object and a derived PDIST predicate.

(35) X-e'-in-q'ete-la' oxi' ak'wal-a'.
CP-A3p-E1s-hug-PDIST three child-PL
I hugged three children individually (many times).

(36) $in^{\textbf{ag}} \rightsquigarrow \lambda \mathcal{P}_{\epsilon t} \lambda \textbf{e}_\epsilon . \mathcal{P}(\textbf{e}) \wedge \textbf{ag}(\textbf{e}) = \textbf{speaker}$

(37) $PAST \rightsquigarrow \lambda \mathcal{P}_{\epsilon t} . \exists \textbf{e}_\epsilon (\mathcal{P}(\textbf{e}) \wedge \textbf{runtime}(\textbf{e}) < \textbf{now})$

An example

We can now get a full translation for this example

(38) X-e'-in-q'ete-la' oxi' ak'wal-a'.
CP-A3p-E1s-hug-PDIST three child-PL
I hugged three children individually (many times).

(39) $\exists \mathbf{e}_\epsilon (\text{HUG}(\mathbf{e}) \wedge |\{e' \le e : \mathbf{atom}(e')\}| > n \wedge$
$\forall e' \le e(\mathbf{atom}(e') \rightarrow \mathbf{atom}(\mathbf{th}(e'))) \wedge$
$\exists x_e (|\{x' \le x : \mathbf{atom}(x')\}| = 3 \wedge \text{CHILD}(x) \wedge \mathbf{th}(\mathbf{e}) = x) \wedge$
$\text{ag}(e) = \text{speaker} \wedge$
$\text{runtime}(e) < \text{now})$

This analysis captures all of the generalizations.

1. Plural events
2. Obligatory distribution over plural objects
3. Repetition with singular objects
4. Cannot create derived plurals (narrowest scope)

An example

(40) $\exists \mathbf{e}_\epsilon(\mathrm{HUG}(\mathbf{e}) \wedge |\{e' \le e : \mathbf{atom}(e')\}| > n \wedge$
$\forall e' \le e(\mathbf{atom}(e') \to \mathbf{atom}(\mathbf{th}(e'))) \wedge$
$\exists x_e(|\{x' \le x : \mathbf{atom}(x')\}| = 3 \wedge \mathrm{CHILD}(x) \wedge \mathbf{th}(\mathbf{e}) = x) \wedge$
$\mathrm{ag}(e){=}\mathrm{speaker} \wedge$
$\mathrm{runtime}(e){<}\mathrm{now})$

▸ There must a plurality of events due to the condition $|\{e' \le e : \mathbf{atom}(e')\}| > n$

▸ As required, this is false if I hugged any subset of the children as a group. The reason is that there would be an atomic subevent $\mathbf{e}'$ of $\mathbf{e}$ whose image under $\mathbf{th}$ would be non-atomic.

▸ We allow for repetition with a singular object because, as a function, it is fine for theta-roles to map multiple events to the same individual.

▸ The translation also correctly predicts that PDIST can't create derived plurals. There is no way for $\forall$ over events to interact with an object quantifier—distributivity is encapsulated.

Alternatives, Lessons, and Extensions

Alternatives, Lessons, and Extensions

The analysis works, but let's consider other alternatives, lessons, and extensions.

Alternatives, Lessons, and Extensions

This analysis makes fairly detailed use of theta-roles. Is there reason to believe that theta-roles are at play and not something more syntactic like the argument position.

- ▶ For instance, perhaps the pluractional distributivity operator composes with the whole VP and not the V, and so it scope over the object, yet not the subject.

Alternatives, Lessons, and Extensions

Evidence that we are actually dealing with theta-roles comes with passives. Recall that PDIST can't force a distributive interpretation of the subject.

(41) X-∅-qa-tun-ula' ri q'ul.
      CP-A3s-E1p-fold-PDIST the blanket
      #We each folded the blanket.

Crucially, it can target passivized subjects:

(42) X-e-pitz'-ilä-x. \ COM-A3p-squeeze-**la'**-PAS \ 'They were squeezed individually.'

This suggests that PDIST does not care about grammatical subjecthood—i.e., the syntactic position of arguments—but thematic notions of argumenthood.

  ▸ Our analysis captures this by treating PDIST as sensitive to the verb's theme theta-role.

Alternatives, Lessons, and Extensions

Recognizing the importance of theta-roles to the analysis raises some important compositional issues.

- ▶ In this account the pluractional is an event modifier and is lexically specified to target the verb's theme theta-role.
- ▶ Another option would be to compositionally target the verb's theme role.

(43) PDIST $\leadsto \lambda\Theta_{\epsilon e}\lambda\mathcal{P}_{\epsilon t}\lambda x\lambda\mathbf{e}_\epsilon.$
$P(e) \wedge \Theta(e) = x \wedge$
$|\{e' \leq e : \mathbf{atom}(e')\}| > n \wedge$
$\forall e' \leq e(\mathbf{atom}(e') \to \mathbf{atom}(\Theta(e')))$

This analysis is quite clean and allows us to distinguish pluractionals from adverbial event modifiers, but I'm not sure how to argue for it. I think this kind of analysis would be really important to explore.

Alternatives, Lessons, and Extensions

Whether or not pluractional distributivity is an event-modifier or a theta-role modifier, it is clearly operating in the VP.

- Is this VP the share or the key?
- According to the analysis here, it is the key!

Recall that:

- **Key:** The thing we break up into parts.
- **Share:** The thing the parts individually participate in.
- **Map:** The relation between key and share

The crucial subformula is: $\forall e' \leq e(\mathbf{atom}(e') \rightarrow \mathbf{atom}(\Theta(e')))$

- The key is the event argument, which we break into atomic parts.
- The share is the theme argument, whose atomic parts are linked up with the parts of the key.
- The map is the theme theta role.

Alternatives, Lessons, and Extensions

This is fairly cool. In most other cases, distributive operators that enforce a distributive interpretation of an arugment have that argument as key.

- ▶ This makes sense because the key is what we are breaking into parts.
- ▶ The way that PDIST gets around this is that the share involves an **atom** predicate over individuals.
- ▶ The result is a kind of simultaneous distribution over both event and individual arguments.

Alternatives, Lessons, and Extensions

I strongly believe that this kind of distributivity should be canonical for pluractionals. Why? Because if the verb is the share and the distributive operator is strong, then while a plurality of events can be produced, the verb is not a **predicate** of plural events, as we may want a pluractional verb to be.

(44) The girls each left.

(45) $\sigma x.\text{GIRL}(x) \wedge \forall x' \leq x[\textbf{atom}(x) \rightarrow \exists e[\text{LEFT}(e) \wedge \textbf{ag}(e) = x]]$

If there is more than one girl, then there will a distributive interpretation of the subject and there will be a plurality of events, but I don't want to call this a distributive pluractional—the verb is not a predicate of pluralities. If it were, each girl would have to leave many times:

(46) $\sigma.x\text{GIRL}(x) \wedge \forall x' \leq x[\textbf{atom}(x) \rightarrow \exists e[\text{LEFT-}\textbf{plrc}(e) \wedge \textbf{ag}(e) = x]]$

In contrast, if the VP is the key in a distributive construction, the verb can naturally accept a plural event in support of distributivity.

(47) $\sigma e.\text{VERB-}\mathbf{plrc}(e) \wedge \rightarrow \forall e' \leq e[\mathbf{atom}(e) \rightarrow \ldots]$

Alternatives, Lessons, and Extensions

Finally, let's consider the question of whether PDIST is merely distributive, or whether it imposes some additional constraints on its key / share.

- ▶ With plural objects, it appears that no additional constraints on imposed. It just looks like we have distributivity.

(48) X-e'-in-kam-ala' ri sanik.
     CP-A3p-E1s-kill-PDIST the ant
     I killed the ants individually.
     False if I killed any subset of the ants simultaneously

Alternatives, Lessons, and Extensions

With singular objects, though, we get additional spatial readings.

- ▶ Various locations of the theme.

(49) X-in-tik-ila' jun che'.
      CP-E1s-plant-PDIST a tree
      I planted a tree various places

- ▶ Various parts of the theme.

(50) X-in-k'ut-ula' jun kem.
      CP-E1s-show-PDIST a weaving
      I showed various parts of the weaving.

Alternatives, Lessons, and Extensions

What do we want to say here? In the case of this example it looks like the distributive pluractional is imposing extra constraints on its share.

(51)   X-in-k'ut-ula' jun kem.
       CP-E1s-show-PDIST a weaving
       I showed various parts of the weaving.

That is, each part of the key (each event), must be mapped, not to an atomic part of the theme, but to spatially distinct part of the theme—e.g.,

- Every atomic event in the plural event of showing is mapped to a part of the weaving by the theme relation.
- No two parts of the weaving that are mapped to by the theme relation occur in the same location.

Alternatives, Lessons, and Extensions

Under this account, the distributive pluractional would be like pluractional adverbial like *one by one*, which are distributive over the individual argument, but in addition say that the distributive share has to be arranged in a certain way.

(52) The students left one by one.

- ▸ Every atomic student participates in their own leaving event.
- ▸ Not to leaving events that a student participated in take place at the same time.

Alternatives, Lessons, and Extensions

This is quite nice and makes the verbal pluractional look like the pluractional adverbial *one by one*.

- ▶ I believe, though, that this is not what we want for the pluractional.
- ▶ Why? We don't need it.

Note that for eventive verbs, if the location of theme of two events is different, then the location the event took place in are different.

- ▶ We can more simply capture data like the following by imposing constraints on the key.

(53) X-in-k'ut-ula' jun kem.
CP-E1s-show-PDIST a weaving
I showed various parts of the weaving.

(54) X-in-tik-ila' jun che'.
CP-E1s-plant-PDIST a tree
I planted a tree various places

Alternatives, Lessons, and Extensions

This new theory would say for an example like the following:

(55) X-in-k'ut-ula' jun kem.
     CP-E1s-show-PDIST a weaving
     I showed various parts of the weaving.

- each atomic event that in the plural event that satisfies the pluractional k'utula' is mapped to an atomic weaving—that is, in each pluractional subevent I show you the weaving
- moreover, no two atomic events that are part of the plural event occur in the same space—thus, the showings are of different parts of the weaving.

Alternatives, Lessons, and Extensions

The same works for these other examples with spatial readings:

(56) X-in-tik-ila' jun che'.
CP-E1s-plant-PDIST a tree
I planted a tree various places

- ▸ each atomic event that in the plural event that satisfies the pluractional tikila'
  is mapped to an atomic tree—that is, in each pluractional subevent I plant
  the same tree.

- ▸ moreover, no two atomic events that are part of the plural event occur in the
  same space—thus, I must plan the tree in different locations.

Alternatives, Lessons, and Extensions

This works for these examples, but what about cases like the following where we never talked about a spatial reading and instead distributive was over individuals.

(57) X-e'-in-kam-ala' ri sanik.
CP-A3p-E1s-kill-PDIST the ant
I killed the ants individually.
False if I killed any subset of the ants simultaneously

Well, in this cases, I think the spatial aspect of the pluractional is taken care of by the fact that different individuals occupy different spaces by definition.

- I cannot kill every any in literally the same space beacause even the dead ones occupy some space that the live ones are not in.

Alternatives, Lessons, and Extensions

The Kaqchikel pluractional marker is thus a distributive operator with the following properties:

- It has an event argument key
- Its share is the atomic parts of an individual
- Its map is a theta role (here the theme)
- It has a secondary spatial commonent it imposes on the key

How common is this kind of pluractional across languages?

How common is this kind of pluractional across languages?

I believe this kind of pluractional, based on theta roles and including an additional constraint on the event-key is pretty common.
Consider some examples:

How common is this kind of pluractional across languages?

Here is the Yup'ik distributive postbase:

<u>Argument plural only: Yup'ik distributive postbase (Jacobson 1984:542)</u>

tekite- 'to arrive'   tekitequut 'they are leaving one after another'

nere- 'to eat'   ner'qui 'he is eating them one after another'

Assuming the event argument is key as in Kaqchikel,

- ▸ The share is the atomic parts of an arugment (though it looks like we cannot have purely atomic arguments).
- ▸ The map is either the theme **or agent** theta roles
- ▸ There is an additional temporal constraint on the key.

How common is this kind of pluractional across languages?

Argument and spatial plural: Evenki distributive (Nedjalkov 1997:251):

d'ava- 'take/seize'

d'ava-ty- 'take/seize several objects one by one'

lo:van- 'hang (meat or fish for drying)'

lo:vat- 'hang (pieces of meat or fish for drying) here and there'

Assuming the event argument is key as in Kaqchikel

- ▸ The share is the atomic parts of an arugment (though it looks like we cannot have purely atomic arguments).
- ▸ The map is the theme theta role.
- ▸ There is an additional spatial **or temporal** constraint on the key.

How common is this kind of pluractional across languages?

In sum, I think across languages we see the following kind of distributive pluractionality that varies along a few parameteres:

- The key is the event.
    - languages can pick whether they add additional temporal or spatial variation constraints on the key events.
- The share is the atomic parts of an argument.
    - languages can pick whether the argument itself is allowed to be atomic.
- The map is a theta role.
    - language can pick which theta roles, but there seems to be a preference typologically for themes.

Dependent pluractionality in Piipaash (Yuman)*

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1 Introduction

Yuman is a family of Indigenous languages spoken in Arizona, Southern California, Baja California, and Northern Sonora.
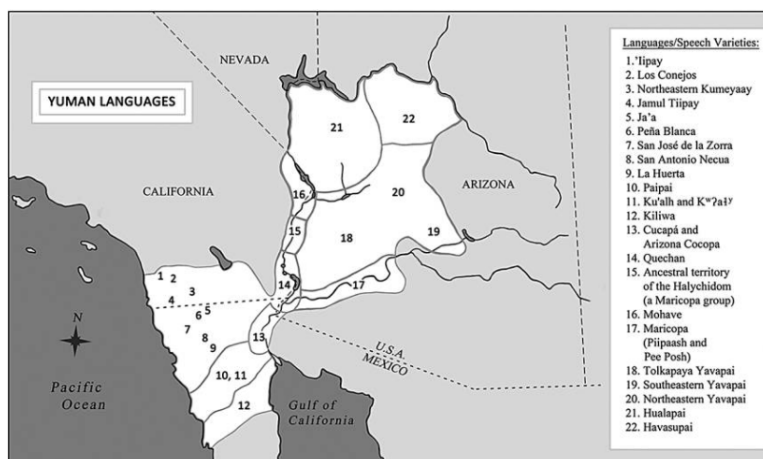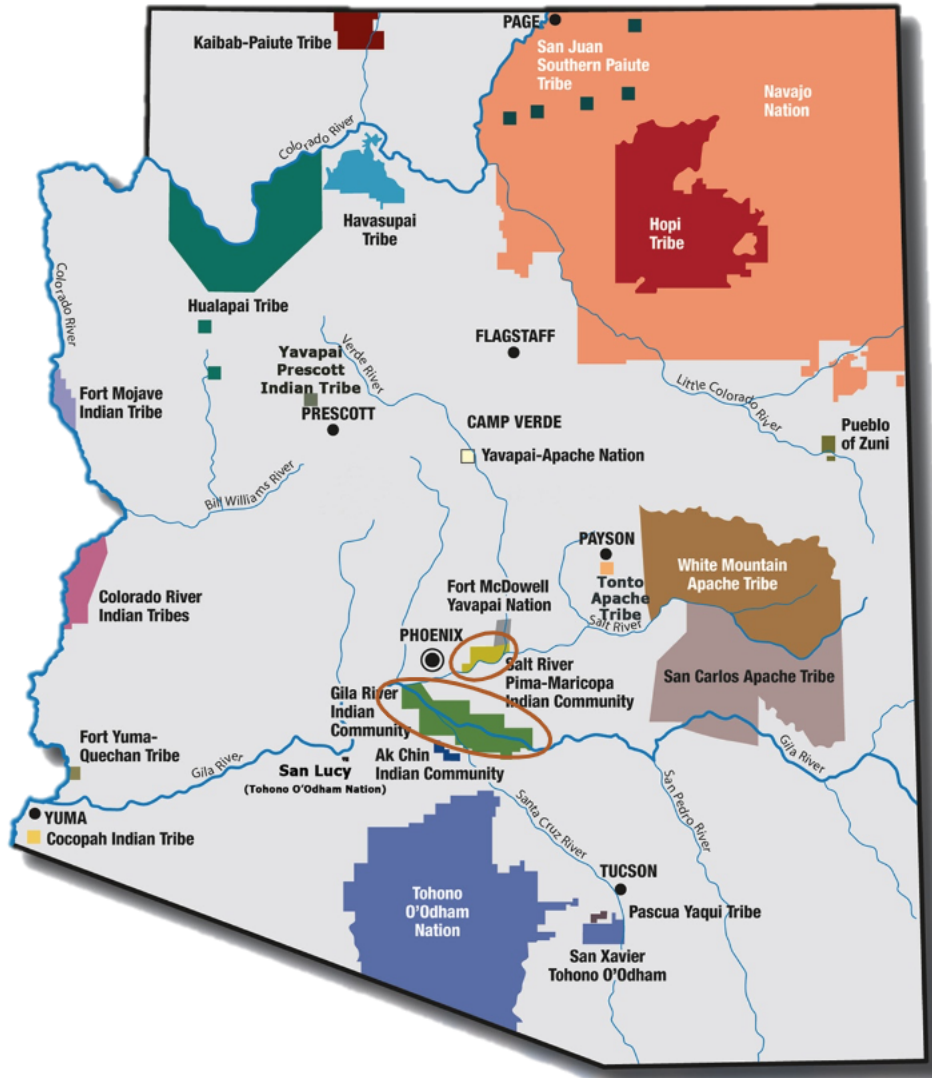

Miller 2018, p. 387

Piipaash is an Indigenous language spoken in Arizona in two communities: Salt River Pima-Maricopa Indian Community (SRPMIC) and Gila River Indian Community (GRIC) along side the Akimel O'odham community, see circled in (3) for map of both nations.

Piipaash has what, at first pass, look like standard dependent indefinites (e.g., Balusu 2006; Farkas 1997; Henderson 2014).

- In the following example, the affix *-xper-*, traditionally glossed 'each', occurs on a numeral that co-varies in the scope of a distributively interpreted subject—i.e., for each of Pam and Heather there is a distinct set of three pieces of bread she ate.

(1)　　Pam-sh　Heather-m　uudav-k　　　paan　xmuk-**xper**-m mash-k
　　　　Pam-NOM Heather-ASC accompany-SS bread three-**each**-DS eat.DU-REAL
　　　　'Pam and Heather each ate three pieces of bread.'　　　　　　　(Gordon, 1986, p. 99)

Looking more broadly we see that *-xper-* has a wider distribution than markers of dependent indefinites in other languages discussed in the literature, and this introduces puzzles.

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Puzzle 1: (1) shows that *-xper-* can mark dependent numerals. It can also mark verbs to yield the same effect. In (2) *-xper-* appears on *tuuwamp* 'turn' and marks the event argument as dependent. It must co-vary in the scope of the subject—i.e., for each there is a distinct event of turning it.

(2)   mat-cham-k   kwnyminy-m tuuwamp-**xper**-k
           REFL-all-SS   different-DS   turn.PL-**each**-REAL
           They all turned it around separately   (Gordon, 1986, p. 144)

How do we account for this apparently cross-categorical effect in a unified way, given that most previous accounts of dependent indefinites in languages like Telugu, Hungarian, Kaqchikel, etc., involve morphology restricted to numerals / indefinite quantifiers? (though see Pasquereau (2019, 2021))

Puzzle 2 (Gil's Puzzle): In Gil's 1982 dissertation he correctly notes that *-xper-* marks distributive shares (i.e., expressions that co-vary in the scope of the distributive operator) across a variety of expressions.

- In that same work, Gil also notes an apparent counterexample to this generalization, which he never solves.

- In particular, *-xper-* can appear on certain coordinations, where the coordinated nominals are interpreted as the distributive key.

(3)   John-sh Bill-sh nyi-dush-**xper**-k   'ii xmok-m   paaysh-k
           John-NOM Bill-NOM PL.OBJ-be.DU-**each**-SS stick three.SG-DS carry.DU-REAL
           John and Bill each carried three sticks.   (Gil, 1982, p. 281, ex. 35c)

Here the existential verb, embedded under the subject, bears *-xper-*.

- Such examples disturbed Gil because *-xper* is inside the subject DP, yet this sentence has a similar interpretation as (1), where *-xper-* marks the object DP.

- We should only mark the latter if *-xper-* marked expressions co-varying under a distributively interpreted expression, not distributively interpreted expressions themselves.

Solution: *-xper-* involves a novel kind of pluractionality that we dub *dependent pluractionality*

- In particular,

  - While in most previously discussed languages the relevant morphology marks an **individual variable** as dependent (i.e., the variable quantified over by a numeral or indefinite).

  - In Piipaash, *-xper-* marks an an **event variable** as dependent.

- We immediately solve Puzzle 1.

- What accounts for this wide distribution of *-xper* compared to dependent indefinites in other languages is that in Piipaash, a wide variety of expressions are verbal, including numerals, coordination, etc., and have an event argument.

- A simple extensions solves Gil's puzzle.

  - If *-xper-* marks dependent pluractionality, it is not marking the nominal in (52), but the verb embedded under that nominal.

  - Thus, it can still be a species of share-marking where the distributively interpreted nominal subject has two shares (i) the *-xper-*marked VP it embeds as a relative clause, and (ii) the main clause VP.

2 Presuppositions about post-suppositions

Henderson 2014 develops an account of dependent in indefinites in the Mayan language Kaqchikel (and other languages) based on the notion of post-suppositions.

(4) K-onojel x-Ø-ki-kanöj   ju-jun   wuj.
     E3p-all   CP-A3s-E3p-search-SS one-RED book
     'All of them looked for a book (and at least two books were looked for).'
     ∗'There is a book and all of them looked for it.'

The backdrop for the account is a version of Dynamic Plural Logic (DPlL) in van den Berg 1996 that has been stripped to its bare essentials.

- Like Dynamic Predicate Logic (Groenendijk and Stokhof, 1991), DPlL formulas are binary relations between variable assignments, which we can think of as input and output contexts.

- That is, a formula ϕ is true relative to g just in case there is an assignment h such that the result of updating g with ϕ is h. W

- here DPlL departs from Dynamic Predicate Logic is that instead of single variable assignments, formulas are interpreted relative to sets of variable assignments $\langle G, H \rangle$ (van den Berg, 1996; Brasoveanu, 2008; Nouwen, 2003, among others).

A set of assignments can be represented as a matrix. The columns of a matrix, like that in ((5)), represent variables (or discourse referents).

(5)

H	...	x	y	...
h_1	...	$entity_1$	$entity_4$	...
h_2	...	$entity_2$	$entity_4$	...
h_3	...	$entity_3$	$entity_4$	...
...	...	...	...	...

Brasoveanu 2011 calls the plurality of individuals stored in x above an EVALUATION PLURALITY, in contrast to a DOMAIN PLURALITY, which is a non-atomic entity (or group-entity) in the domain. I will continue to use this terminology in what follows.

Why should we move to a dynamic semantics with plural variable assignments?

(6) Each student brought a^i lunchbox. They put them$_i$ on the shelf.

Using the DPlL framework makes sense for our purposes, then, because it allows indefinites to yield plural discourse referents when they are embedded in quantificational domains.

- In this account, dependent indefinites mandate plural discourse reference for the variable they bind, and they do so in such a way that they must be in the scope of a distributive operator.

We thus get the the formal typology of indefinite plurality in Figure 2, which Kaqchikel completely instantiates.

	Domain Singular	Domain Plural
Evaluation Singular	*jun*	*oxi'*
	one	three
Evaluation Plural	*ju-jun*	*ox-ox*
	one-RED	three-RED

Figure 1: Typology of indefinite plurality

- Our big idea is that Piipaash does the exact same thing, but in the domain of events, rather than the domain of individuals.

	Domain Singular	Domain Plural
Evaluation Singular	verb	pluractional verb
Evaluation Plural	*xper*-marked verb	*xper*-marked pluractional verb

Figure 2: Typology of verbal plurality

Basic tech:
Atomic formulas are tests (they only pass on input contexts that satisfy them). Note that they are interpreted distributively with respect to assignments in H.

(7) $[\![R(x_1, \ldots, x_n)]\!]^{\langle G,H \rangle} = \mathbb{T}$ iff $G = H$ and $\forall h \in H, \langle h(x_1), \ldots, h(x_n) \rangle = \mathfrak{I}(R)$

We clearly need expressions to manipulate these two kinds of pluralities.

- Domain-level cardinality predicates—**one**(x), **two**(x), etc.—distributively check the cardinality of the set of atomic parts of an individual.

(8) $\llbracket\textbf{two}(x)\rrbracket^{\langle G,H\rangle} = \mathbb{T}$ iff $G = H$ and for all $h \in H$,
$|\{x' : x' \leq h(x) \wedge \textbf{atom}(x')\}| = 2$

- Essentially, given G, check whether $|\textbf{atoms}(g_1(x))| = 2$, and $|\textbf{atoms}(g_2(x))| = 2$, etc.

- We also have tests for evaluation-level cardinality. Essentially, given G, they check the cardinality of $\{ g_1(x), g_2(x), g_3(x),... \}$

(9) $G(x) := \{g(x) : g \in G\}$

(10) $\llbracket x = n \rrbracket^{\langle G,H\rangle} = \mathbb{T}$ iff $G = H$ and $|H(x)| = n$

Dynamic conjunction is defined as relation composition.

(11) $\llbracket \phi \wedge \psi \rrbracket^{\langle G,H\rangle} = \mathbb{T}$ iff there is a K s.t. $\llbracket \phi \rrbracket^{\langle G,K\rangle} = \mathbb{T}$ and $\llbracket \psi \rrbracket^{\langle K,H\rangle} = \mathbb{T}$

Quantification proceeds via pointwise manipulation of assignment functions. We overload the notation [x] to define random assignment in the object language.

(12)   Random assignment: $\llbracket [x] \rrbracket^{\langle G,H\rangle} = \mathbb{T}$ iff $G[x]H$, where

a. $G[x]H := \begin{cases} \text{for all } g \in G, \text{ there is a } h \in H \text{ such that } g[x]h \\ \text{for all } h \in H, \text{ there is a } g \in G \text{ such that } g[x]h \end{cases}$, and

b. $g[x]h$ iff for any variable v, if $v \neq x$, then $g(v) = h(v)$

Verbs have an event argument, which is existentially closed by default. They are connected to their arguments via theta-roles (AG, TH, etc.), which are distinguished functional relations from the domain of events to the domain of individuals.[1]

An example:
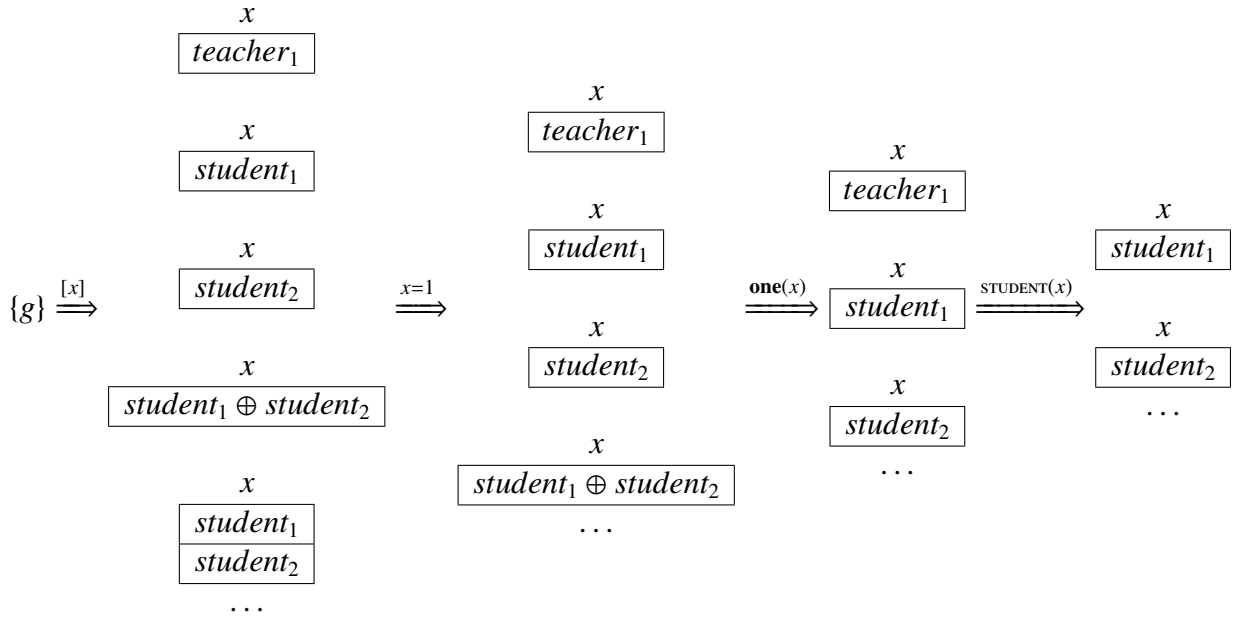Putting things together, the sentence 'A student danced' is translated as in ((13)).

(13)   A student danced $\rightsquigarrow$
$\exists x[x = 1 \wedge \textbf{one}(x) \wedge \text{STUDENT}(x)](\exists e(e = 1 \wedge \text{DANCE}(e) \wedge \text{AG}(e, x)))$

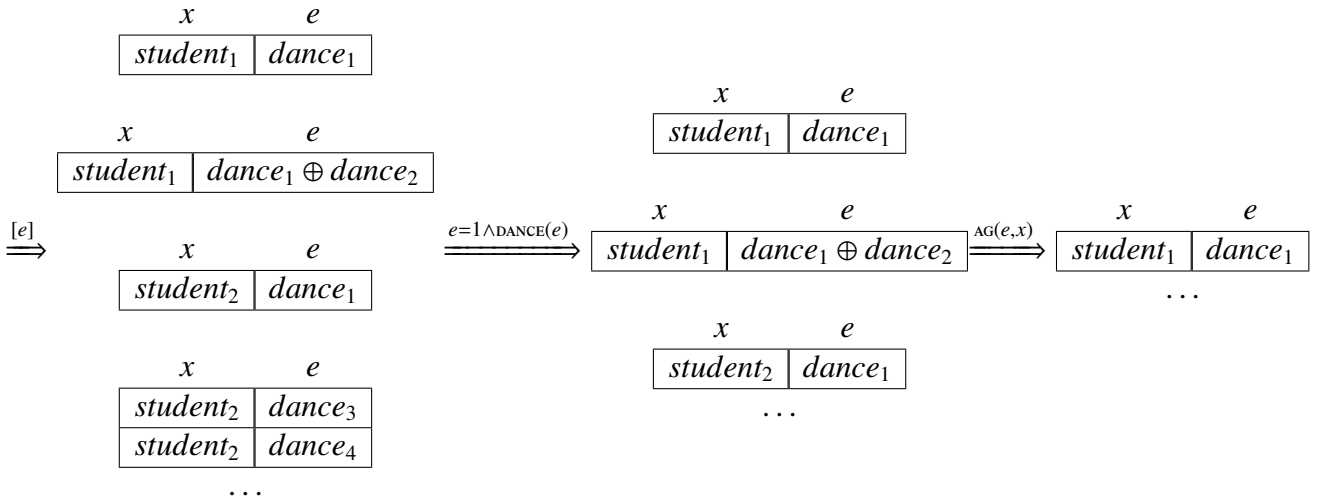The formula in example ((13)) just abbreviates the dynamic version in ((14)).

(14) $[x] \wedge x = 1 \wedge \textbf{one}(x) \wedge \text{STUDENT}(x) \wedge [e] \wedge e = 1 \wedge \text{DANCE}(e) \wedge \text{AG}(e, x)$

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[1]I also assume that these theta-roles, in addition to basic lexical relations (SEARCH, EAT, STUDENT etc.), are cumulatively closed by default, though I suppress the common star notation for readability. That is, we assume that all theta-roles and n-ary lexical relations R are always $**R$, where $**R$ is the smallest set such that $R \subseteq **R$ and if $\langle a_1, ..., a_n \rangle \in **R$ and $\langle b_1, ..., b_n \rangle \in **R$, then $\langle a_1 \oplus b_1, ..., a_n \oplus b_n \rangle \in **R$. Note that domain-level cardinality predicates are not to be interpreted cumulatively, just like the metalanguage predicate **atom**, which is why they will also be marked in bold throughout.

Suppose that our input context is a singleton assignment assigning some value to every variable:
$[x] \wedge x = 1 \wedge \textbf{one}(x) \wedge \text{STUDENT}(x) \wedge [e] \wedge e = 1 \wedge \text{DANCE}(e) \wedge \text{AG}(e, x)$

$$\{g\} \overset{[x]}{\Longrightarrow} \quad \overset{x=1}{\Longrightarrow} \quad \overset{\textbf{one}(x)}{\Longrightarrow} \quad \overset{\text{STUDENT}(x)}{\Longrightarrow}$$

First step — $[x]$:

x
$teacher_1$

x
$student_1$

x
$student_2$

x
$student_1 \oplus student_2$

x
$student_1$
$student_2$

$\ldots$

Second step — $x=1$:

x
$teacher_1$

x
$student_1$

x
$student_2$

x
$student_1 \oplus student_2$

$\ldots$

Third step — $\textbf{one}(x)$:

x
$teacher_1$

x
$student_1$

x
$student_2$

$\ldots$

Fourth step — $\text{STUDENT}(x)$:

x
$student_1$

x
$student_2$

$\ldots$

The next block begins by introducing an event e. Just as before, potential outputs could store in e a non-atomic event or an evaluation plurality.

$$\overset{[e]}{\Longrightarrow} \quad \overset{e=1 \wedge \text{DANCE}(e)}{\Longrightarrow} \quad \overset{\text{AG}(e,x)}{\Longrightarrow}$$

First step — $[e]$:

x	e
$student_1$	$dance_1$

x	e
$student_1$	$dance_1 \oplus dance_2$

x	e
$student_2$	$dance_1$

x	e
$student_2$	$dance_3$
$student_2$	$dance_4$

$\ldots$

Second step — $e=1 \wedge \text{DANCE}(e)$:

x	e
$student_1$	$dance_1$

x	e
$student_1$	$dance_1 \oplus dance_2$

x	e
$student_2$	$dance_1$

$\ldots$

Third step — $\text{AG}(e,x)$:

x	e
$student_1$	$dance_1$

$\ldots$

(15)   Truth: a formula ϕ is true relative to an input context G iff there is an output set of assignments H s.t. $[\![\phi]\!]^{\langle G,H \rangle} = \mathbb{T}$.

In the illustrated examples that follow, we will only represent one typical path through the graph.

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$[x] \wedge x=1 \wedge \mathbf{one}(x) \wedge \text{STUDENT}(x) \wedge [e] \wedge e=1 \wedge \text{DANCE}(e) \wedge \text{AG}(e,x)$

x	e
$student_1$	$dance_1$

Because distributive quantifiers license dependent indefinites, let's consider how universal quantification is treated in DPlL. This will lay the foundation for analyzing how dependent indefinites are licensed in their scope.

- We decompose universal quantification into a maximization operation over the restrictor and a distributive operator over the nuclear scope (Brasoveanu, 2008).

- That is, $\forall x[\phi](\psi)$ abbreviates $\mathbf{max}^x(\phi) \wedge \delta(\psi)$.

(16) $[\![\mathbf{max}^x(\phi)]\!]^{\langle G,H \rangle} = \mathbb{T}$ iff $[\![[x] \wedge \phi]\!]^{\langle G,H \rangle} = \mathbb{T}$ and

    a.   There is no H' such that $H(x) \subsetneq H'(x)$ and $[\![[x] \wedge \phi]\!]^{\langle G,H' \rangle} = \mathbb{T}$

(17) $[\![\delta(\phi)]\!]^{\langle G,H \rangle} = \mathbb{T}$ iff there exists a partial function $\mathcal{F}$ from assignments g to sets of assignments K, i.e., of the form $\mathcal{F}(g) = K$, s.t.

    a. $G = \mathbf{Dom}(\mathcal{F})$ and $H = \bigcup \mathbf{Ran}(\mathcal{F})$
    b.   for all $g \in G$, $[\![\phi]\!]^{\langle \{g\}, \mathcal{F}(g) \rangle} = \mathbb{T}$

Consider an example like *'Every boy left'*, whose translation appears in ((18)-(19)).

(18) $\forall x[\text{BOY}(x) \wedge \mathbf{one}(x)](\exists e(e = 1 \wedge \text{LEFT}(e) \wedge \text{AG}(e, x)))$

(19) $\mathbf{max}^x(\text{BOY}(x) \wedge \mathbf{one}(x)) \wedge \delta([e] \wedge e = 1 \wedge \text{LEFT}(e) \wedge \text{AG}(e, x))$
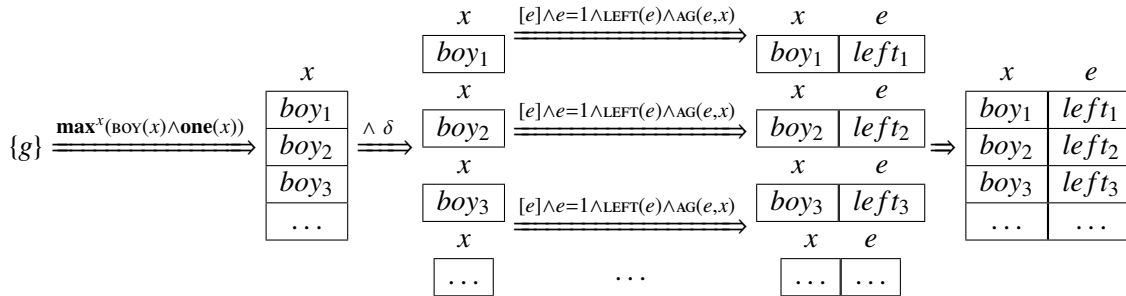
To presage the analysis of dependent indefinites, note that as long as more than one individual in the model satisfies the restrictor, interpreting a universal quantifier can result in evaluation plural discourse referents for indefinites in its scope.

2.1   Dependent indefinites

The heart of the proposal is that dependent indefinites are like simple indefinites, except that they must come to contribute an evaluation plurality from the perspective of the *global* discourse context.

- In this way, dependent indefinites are similar to expressions bearing presuppositions or conventional implicatures.

- Just like these expressions, part of their meaning contributes to the at-issue content, while a second part is interpreted separately.

- The difference is where this secondary content is interpreted. For presuppositions, it must be interpreted relative to the input context, that is, before the at-issue content (van der Sandt, 1992; Kamp, 2001, among others).

- In contrast, we argue that the cardinality constraint of dependent indefinites is a post-supposition interpreted *after* the at-issue update.

- In essence, this allows the dependent indefinite to be interpreted in-situ, but take a global perspective on the environment in which it is interpreted.

Post-suppositions are not a new class of meanings. They are discussed in Constant 2012; Farkas 2002; Lauer 2009, though Brasoveanu 2012 gives the most thorough formal treatment, which we will follow closely.

- The core definition is that in ((20)), where post-suppositions are marked via an $\overline{\text{overline}}$

(20) $[\![\overline{\phi}]\!]^{\langle G[\zeta], H[\zeta']\rangle} = \mathbb{T}$ iff ϕ is a test, $G = H$ and $\zeta' = \zeta \cup \{\phi\}$.[2]

(21)   Truth: ϕ is true relative to an input context $G[\emptyset]$ iff there is an output set of assignments H and a (possibly empty) set of tests $\{\psi_1, \ldots, \psi_m\}$ s.t. $[\![\phi]\!]^{\langle G[\emptyset], H[\{\psi_1,\ldots,\psi_m\}]\rangle} = \mathbb{T}$ and $[\![\psi_1 \wedge \ldots \wedge \psi_m]\!]^{\langle H[\emptyset], H[\emptyset]\rangle} = \mathbb{T}$.

For a concrete example, consider a formula like $\overline{\phi} \wedge \psi$, where ψ contains no post-suppositions.

(22) $[\![\overline{\phi} \wedge \psi]\!]^{\langle G[\emptyset], H[\{\phi\}]\rangle} = \mathbb{T}$ iff there is a $K[\zeta]$ such that
   a. $G = K$
   b. $\zeta = \emptyset \cup \{\phi\}$
   c. $[\![\psi]\!]^{\langle K[\{\phi\}], H[\{\phi\}]\rangle} = \mathbb{T}$
   d. $[\![\phi]\!]^{\langle H[\emptyset], H[\emptyset]\rangle} = \mathbb{T}$

Recall that plain indefinites contribute variables that are evaluation singular in their local context.

(23)   one ϕ is ψ $\leadsto \exists x[x = 1 \wedge \mathbf{one}(x) \wedge \phi](\psi)$

Where dependent indefinites differ is that they place the post-suppositional test $\overline{x > 1}$ on the variable they bind.[3]

(24)   one$_{dependent}$ ϕ is $\psi \leadsto \exists x[\overline{x > 1} \wedge \mathbf{one}(x) \wedge \phi](\psi)$

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[2] ϕ is a test just in case for any sets of assignments G and H and any sets of formulas ζ and ζ', if $[\![\phi]\!]^{\langle G[\zeta], H[\zeta']\rangle} = \mathbb{T}$, then $G = H$ and $\zeta = \zeta'$.

[3] For dependent numerals, replace **one** in ((24)) with the appropriate cardinality predicate (**two**, **three**, etc.).

To see the translation in ((24)) in action, consider example ((25)), which has the reduplicated form of the indefinite *jun* 'one'.

(25)   Chi-ki-jujunal ri tijoxel-a'   x-Ø-ki-q'etej   ju-jun   tz'i'.
        P-E3p-each   the student-PL CP-A3s-E3p-hug one-RED dog
        'Each of the students hugged a dog.' and FALSE if they all hugged the same dog.

Example ((26)) gives a translation of ((25)) using the ∀/∃ shorthand making relative scope easier to see.

(26) $\forall x[\mathbf{one}(x) \wedge \text{STUDENT}(x)]$
                $(\exists y[y > 1 \wedge \mathbf{one}(y) \wedge \text{DOG}(y)]$
                        $(\exists e(e = 1 \wedge \text{HUG}(e) \wedge \text{AG}(e, x) \wedge \text{TH}(e, y))))$

Because the dependent indefinite's post-supposition is evaluated globally, ((26)) is equivalent to ((27)), where $y > 1$ takes widest scope.

(27) $\mathbf{max}^x(\mathbf{one}(x) \wedge \text{STUDENT}(x)) \wedge \delta([y] \wedge \mathbf{one}(y) \wedge \text{DOG}(y) \wedge [e] \wedge e = 1 \wedge \text{HUG}(e) \wedge \text{AG}(e, x) \wedge \text{TH}(e, y)) \wedge y > 1$
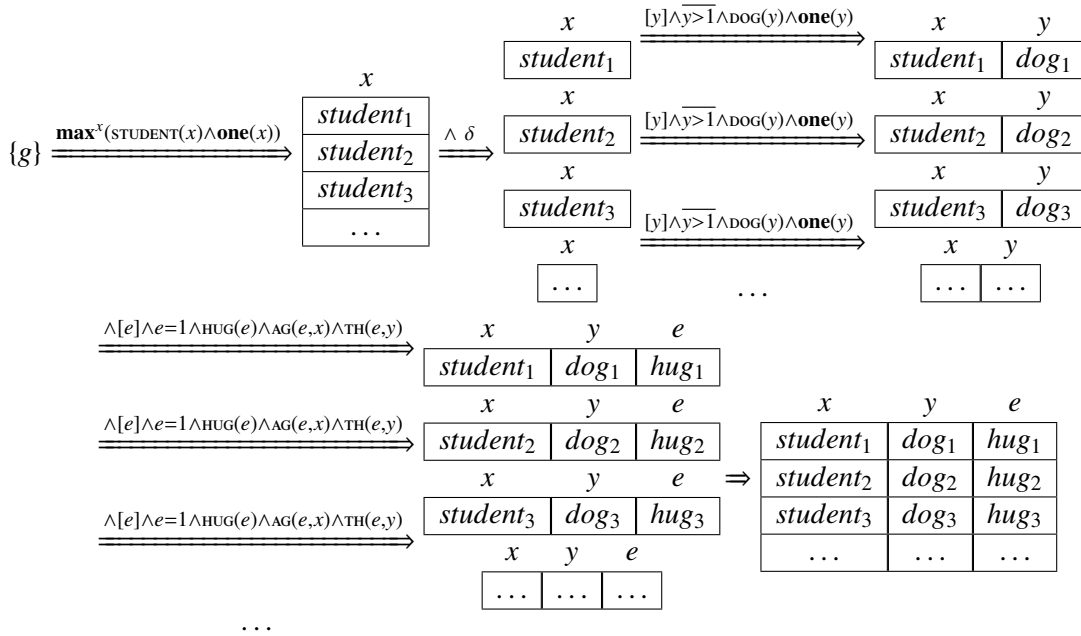
The figure above illustrates how the analysis hinges on treating the test $\overline{y > 1}$ as a post-supposition.

- If it were interpreted locally, that is, in the scope of the distributivity operator, we would have to satisfy $y > 1$ as we interpret the nuclear scope relative to each singleton assignment storing an atomic student.

- That is, we would incorrectly require each student to hug at least two dogs. Instead, the test $\overline{y > 1}$ is interpreted last, relative to the final matrix above.

Crucially, until the post-suppositional cardinality condition $\overline{y > 1}$ is evaluated, a formula like (**??**) is completely consistent with an output like ((28)).

(28) $\xrightarrow{\textbf{alternative output}}$

x	y	e
$student_1$	dog_1	hug_1
$student_2$	dog_1	hug_2
$student_3$	dog_1	hug_3
$\dots$	$\dots$	$\dots$

- The post-supposition $\overline{y > 1}$ will not be satisfied in an output like ((28)), preventing the indefinite from taking narrow scope, but failing to covary.

It is this same reasoning that prevents a dependent indefinite from taking wide scope. In this case, the variable that the indefinite binds will fail to covary in principle and thus fail to be evaluation plural in the output.
The analysis clearly explains why dependent indefinites take narrow scope and covary when they can, but what is the cause of ungrammaticality when they do not have a quantificational clause-mate?

- This is due to the fact that, by default, other existential quantifiers contribute evaluation singularities.

- In particular, the existential closure of the event argument introduces a variable that is evaluation singular.

- Without a quantificational clause-mate (or a pluractional, as we will see), a theta dependency linking the event and dependent indefinite always fails to hold.

(29) *X-e'-in-q'etej ox-ox ak'wal-a'.
      CP-A3p-E1s-hug three-RED child-PL
      Desired reading: 'I hugged groups of three children.'

As a dependent indefinite, *oxox* contributes the cardinality constraint in the restrictor of the existential quantifier over individuals. It requires the variable x to store an evaluation plurality.

(30) $\exists x[\, \overline{x > 1} \wedge \textbf{three}(x) \wedge \text{CHILD}(x)](\exists e(e = 1 \wedge \text{HUG}(e) \wedge \text{TH}(e, x)))$

(31) $[x] \wedge \overline{x > 1} \wedge \textbf{three}(x) \wedge \text{CHILD}(x) \wedge [e] \wedge e = 1 \wedge \text{HUG}(e) \wedge \text{TH}(e, x)$

If x were evaluation singular, as with a plain indefinite, every $h \in H$ of any set of output assignments satisfying the formula would store the same sum of three children in x. Therefore a theta-role function can hold between e and x.[4]

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[4]Recall that such functions are interpreted distributively with respect to sets of assignments (see definition **??**).

(32)

H	y	e	x	z
h_1	...	hug_1	$child_1 \oplus child_2 \oplus child_3$	...
h_2	...	hug_1	$child_1 \oplus child_2 \oplus child_3$	...
h_3	...	hug_1	$child_1 \oplus child_2 \oplus child_3$	...
...	...	...	...	...

The situation is completely different with ((30)-(31)), as we see graphically below.

(33)

H	y	e	x	z
h_1	...	hug_1	$child_1 \oplus child_2 \oplus child_3$	...
h_2	...	hug_1	$child_4 \oplus child_5 \oplus child_3$	...
h_3	...	hug_1	$child_6 \oplus child_7 \oplus child_8$	...
...	...	...	...	...

Here e is still evaluation singular—every $h \in H$ assigns e to the same event.

- But now the reduplicated numeral requires that at least two $h \in H$ disagree on their assignments to x because it is evaluation plural.

- The result is that no exhaustive theta-role function can hold between e and x because there can be no functional dependency between e and x.

- Unless there is something generating an evaluation plurality of events, like a wide-scoping nominal quantifier, sentences with singular subjects and reduplicated numerals are predicted to be ungrammatical.

Whew...we can get back to Piipaash.

3   *-xper-* as a maker of dependent pluractionality

Our core proposal, developed in this section, is that

- numerals in Piipaash can bear pluractionality (following Pasquereau (2019, 2021)'s work on Seri (isolate))

- *-xper-* is a marker of a novel species of pluractionality, which we call *dependent pluractionality*, on analogy with *dependent indefinites* (e.g., Henderson 2014; Farkas 1997, 2001, among others).

We can run the same kind of analysis for *-xper-* that we saw in Kaqchikel for dependent indefinites, but recognizing that *-xper-* is a pluractional marker (following the analysis of a similarly transcategorial marker in Seri (isolate; Pasquereau 2019, 2021)).

- This means that *-xper-* should count events in output sets of assignments.

- Because events require a counting criterion, we add a parameter to the <-symbol.

- We let the Θ parameter be set contextually (because *-xper-* can target different theta roles, but we could set this compositionally if the pluractional were a theta role modifier.)

(34) $e >_\Theta 1 =_{def} |\{\Theta(e') : e' \in G(e)\}| < 1$
'The variable e stores more than one event across a set of assignment G just in case it stores at least two events that differ on Θ.'

(35) *-xper-* $\rightsquigarrow \lambda V \lambda e[V(e) \wedge \overline{e >_\Theta 1}]$

- Note that counting verbs in this way predicts that *xper*-marked verbs should only involve participant pluractionality, which is the case—i.e., we don't *-xper-* being licensed by adverbial quantifiers over events.

Let's start with the case where *-xper-* targets a main-clause verb. This is the simplest case for the proposed analysis, which we can extend out to all the other cases to provide a unified analysis.

(36) mxaa-sh ashuuvar-xper-k
boys-NOM 3.sang.PL-each-REAL
'Some/the boys each sang.'   (Gil, 1982, p.271 ex. 24)

If we take the stem *ashuuvar* 'sing' to denote a predicate of events, its *-xper-* form would be predicate of events that are evaluation plural.

(37) *ashuuvar-xper-k* $\rightsquigarrow \lambda e[\text{SING}(e) \wedge \overline{e >_{\text{AG}} 1}]$

The result is a verb stem that must be existentially closed before being placed in the scope of a distributive operator. In this way, conditions like $\overline{e >_{\text{AG}} 1}$ act like powerful filters on representations.

- The filter can be met in sentences like (36) because Piipaash allows the covert distributive interpretation of subjects, like the following.

(38) kafe '-sish-k   pastel '-mash-k
coffee 1-drink.DU-ss pie   1-eat.DU-REAL
'We (two) drank coffee and ate pie.'   (Gordon, 1986, p. 116)

This means that (36) can be interpreted as in (39).

(39) $\forall x[x \leq \sigma y.*boy(y) \wedge \textbf{one}(x) \rightarrow \exists e[\text{SING}(e) \wedge \overline{e >_{\text{AG}} 1} \wedge \text{AG}(e) = x]]$
'True just in case for every atomic boy, there is a singing event he is agent of, and at there are at least two such events (with different agents).'

$$\{g\} \xrightarrow{\text{max}^x(\text{\scriptsize BOY}(x)\wedge\textbf{one}(x))}$$
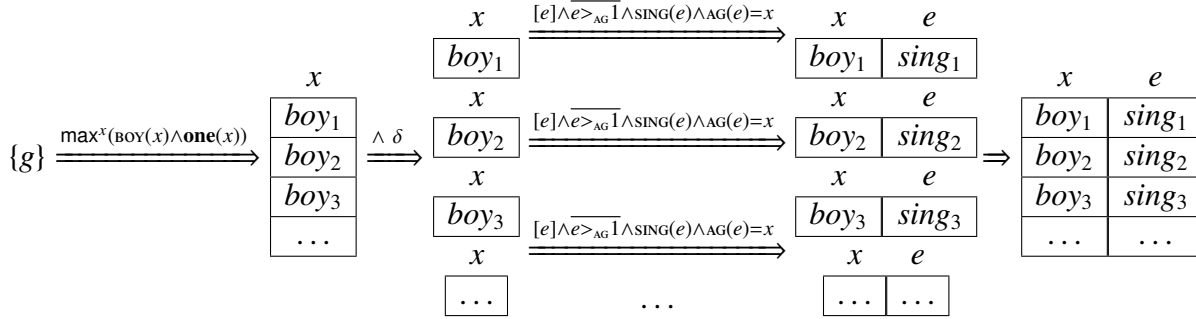
- The universal quantifier introduces a new variable assignment for each restrictor entity—i.e., atomic boy in the sum of *BOY. Each of those assignments is extended with a possibly different e by existential quantification over the event variable allowing $\overline{e >_{\text{\scriptsize AG}} 1}$ to be satisfied.

Note that without an intervening distributive quantifier, a *xper*-marked verb is necessarily false—e.g.,

(40) $\exists e[\text{\scriptsize SING}(e) \wedge \overline{e >_{\text{\scriptsize AG}} 1} \wedge \text{\scriptsize AG}(e) = \sigma y.*\text{\scriptsize BOY}(y)]$

- The problem is that even if e is an ontologically plurality—i.e., the variable assignment maps e to a sum—whose parts are mapped by AG to different boys, it cannot satisfy $\overline{e >_{\text{\scriptsize AG}} 1}$ because $\exists e$ only extends a single variable assignment rather than introducing a plurality of such assignments.

The result is that a main verb marked with -*xper*- must be interpreted in the scope of a distributive operator with existential closure introducing at least two events that scope.

- But? But? Why the runaround? Why not treat -*xper*- as the distributive operator itself?

First, this approach correctly predicts that *xper*-marked verbs should not clash with other bona fide distributivity operator on the distributive key. Consider the following.

(41)   'ny-ku-shiint nyaa   xumar ku-shent '-ashkyet-xper-k
        1-REL-one.PL 1.NOM child   REL-one 1-cut.DIST-each-REAL
        Each of us spanked the child   (Gordon, 1986, p. 144)

- It is perfectly fine for the distributively marked subject *'nykushiint nyaa* 'each of us' to co-occur with a *xper*-marked verb. As we have argued, *xper*-marked verbs, in fact, *must* be in the scope of a distributive operator.

- We explain then why -*xper*- patterns differently from doubling bona fide distributive operators which can produce clashes—e.g., 'Each of us (#each) spanked the child (#each).'

Second, this approach to *-xper-* will permit a unified account when we move to other constructions in which it occurs. In particular, consider the case where *-xper-* marks DP-internal nominal.

(42) Pam-sh Heather-m uudav-k paan xmuk-xper-m mash-k
      Pam-NOM Heather-ASC accompany-SS bread three-each-DS eat.DU-REAL
      'Pam and Heather each ate three pieces of bread.' (Gordon, 1986, p. 99)

There are three critical things to see about this example:

- First, *-xper-* appears on the numeral *xmuk* 'three' inside the nominal constituent headed by *paan* 'bread'.

- Second, the numeral is actually a verb, which we can tell from the fact that is marked DS for switch reference.

- Finally, in this example it is the subject 'Pam and Heather' that is interpreted distributively.

The last point, coupled with the first, shows why treating *-xper-* as a marker of dependent pluractionality is required.

- While it plausible in example like (36) to let *-xper-* compose with the verb and quantify over the subject, a verbal argument, it is hard to imagine how *-xper-*, deeply embedded in an object numeral quantifies over the subject.

- In contrast, the numeral in examples like (42) look almost exactly like dependent numerals in languages like Kaqchikel—i.e., a numeral that must covary in the scope of another expression.

- We say almost because unlike dependent numerals in more familiar languages, in Piipaash, numerals are verbs.

- Ultimately, this supports our analysis of *-xper-* as a kind of pluractionality, namely dependent pluractionality, but we must first understand how verbal numerals could work.
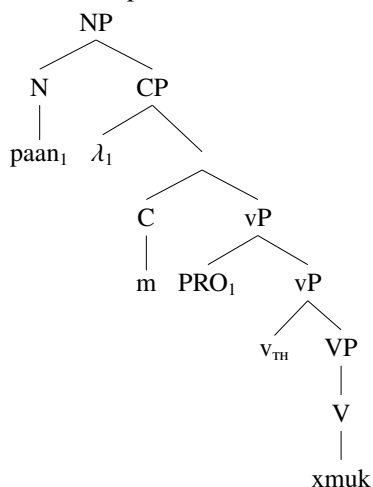
Following Champollion 2016; Kuhn 2019; Pasquereau 2019 we can take numerals to be predicates of event(ualitie)s—events with *n* participants.

(43) *xmuk* $\rightsquigarrow \lambda e[|\text{TH}(e)| = 3]$

We assume the following LF based on work in Seri (Pasquereau, 2019, 2021), itself assuming the analysis of internally-headed relative clauses in Toosarvandani 2014.[5]

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[5]We being loose with our use of the term relative clause in this work. There are a variety of subkinds of relative clauses in Piipaash, and we do not fully understand the syntax of all of them. The constructions we call relative clauses here all involve switch reference subordination.

(44) LF of NP *paan xmukm* "three (pieces of) bread"

```
                    NP
                 ┌───┴────┐
                 N        CP
                 │     ┌───┴────┐
              paan₁    λ₁
                     ┌──────┴──────┐
                     C             vP
                  ┌──┴──┐      ┌────┴────┐
                  m    PRO₁   vP
                            ┌──┴──┐
                           v_TH   VP
                                   │
                                   V
                                   │
                                 xmuk
```

The bottom-line truth conditions of a numerically quantified NP like *paan xmukm* 'three (pieces of) bread' would be as follows:

(45)　*paan xmukm* ⤳ $\lambda x \exists e[|\text{TH}(e)| = 3 \wedge \text{TH}(e) = x \wedge \text{BREAD}(x)]$
　　　'True of bread individuals that number three and participate in an event together.'

These type $\langle et \rangle$ expressions can then be further modified by standard quantifiers, definite articles, etc.

- Note that the numeral does not have existential force. Important for us, bare NPs in Piipaash most often get an existential interpretation—though such NPs are ambiguous with a definite interpretation.

- We assume this existential interpretation numerals often have is due to a null indefinite quantifier.

(46)　$\emptyset_{ind}$ ⤳ $\lambda P \lambda Q \exists x[P(x) \wedge Q(x)]$

We know have all the ingredients to show the dependent numeral effect familiar from languages like Kaqchikel or Hungarian, but through pluractionality.

- Because numerals in Piipaash are event-denoting, we predict that they can be subject to pluractional derivation.

(47)　*paan xmukxperm* ⤳ $\lambda x \exists e[|\text{TH}(e)| = 3 \wedge \text{TH}(e) = x \wedge \overline{e >_{\text{TH}} 1} \wedge \text{BREAD}(x)]$
　　　'True of bread individuals that number three and participate in an event, where that event must co-vary across output assignments.'

If we assume a null indefinite quantifier takes this NP as an argument, we get the following quantificational DP.

(48) $\emptyset_{ind}$ *paan xmukxperm* $\rightsquigarrow \lambda Q \exists x \exists e[|\text{TH}(e)| = 3 \wedge \text{TH}(e) = x \wedge \overline{e >_{\text{TH}} 1} \wedge \text{BREAD}(x) \wedge Q(x)]$

> **Note:** The fact that we have existential interpretation of the DP is what will allow both individuals and, critically, events to co-vary in the scope of some higher quantifier. We predict definite interpretations of nominals embedding *xper*-marked numerals to be infelicitous.

We now have the following VP translation for *eat three-plurc bread*:

(49)   *paan xmukxperm mashk* $\rightsquigarrow \lambda x \exists y \exists e[|\text{TH}(e)| = 3 \wedge \text{TH}(e) = y \wedge \overline{e >_{\text{TH}} 1} \wedge \text{BREAD}(y) \wedge$
   $\exists e'[eat(e') \wedge \text{AG}(e') = x \wedge \text{TH}(e') = y]]$

We are at the crucial step. If the subject of a sentence like (42), namely 'Pam and Heather', were fed as a type e argument to this verb phrase, the result would be infelicitous, a contradiction that could never be true.

- The problem is that there are only existential quantifiers in this sentence, and so $\overline{e >_{\text{TH}} 1}$ is interpreted relative to a single variable assignment, and so cannot be satisfied.

- We must instead have a distributive operator so that the variable e can co-vary in its scope.

- That is, the subject should receive a distributive interpretation, like it, in fact does, in the attested example.

Our final bottom-line truth conditions for a sentence like (42), repeated below, are thus:

(50)   Pam-sh   Heather-m   uudav-k   paan   xmuk-xper-m   mash-k
    Pam-NOM Heather-ASC accompany-SS bread three-each-DS eat.DU-REAL
    'Pam and Heather each ate three pieces of bread.'   (Gordon, 1986, p. 99)

(51) $\forall x[x \leq p \oplus h \wedge \textbf{one}(x) \rightarrow \exists y \exists e[|\text{TH}(e)| = 3 \wedge \text{TH}(e) = y \wedge \overline{e >_{\text{TH}} 1} \wedge \text{BREAD}(y) \wedge \exists e'[eat(e') \wedge$
   $ag(e') = x \wedge th(e') = y]]]$
   'True if for each of Pam and Heather there is an event involving three bread participants y (and there must be at least two such events with different participants in the output), and there is a second event of eating in which she eats y.'
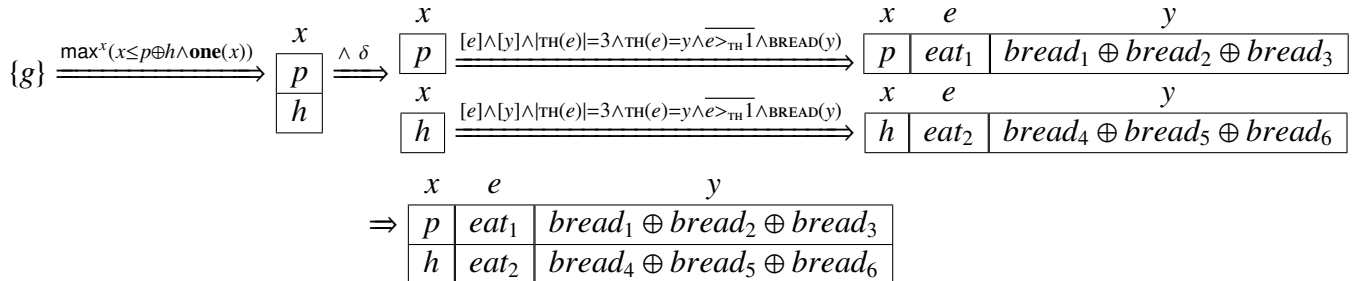
The fact that Piipaash and Kaqchikel both have dependent numerals that have a similar effect on the global truth conditions of the sentences in which they occur, but achieve that effect through different routes is, well, quite beautiful.

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3.1 Solving Gil's puzzle

In Gil's dissertation 1982 he correctly notes that *-xper-* marks distributive shares.

- This follows from our analysis because the post-supposition introduced by *-xper-* can only be satisfied in the scope of a distributive operator.

In that same work, Gil also notes an apparent counterexample to this generalization, which he never solves.

- In particular, *-xper-* can appear on certain coordinations, where the coordinated nominals are interpreted as the distributive key.

(52)   John-sh Bill-sh nyi-dush-**xper**-k   'ii   xmok-m   paaysh-k
       John-NOM Bill-NOM PL.OBJ-be.DU-each-ss stick three.SG-DS carry.DU-REAL
       John and Bill each carried three sticks.   (Gil, 1982, p. 281, ex. 35c)

Here the existential verb, embedded under the subject, bears the *-xper-*.

- Such examples are initially disturbing, and disturbed Gil, because the subject is the distributive key.

Our analysis of *-xper-* as a marker of dependent pluractionality can immediately account for such examples.

- Crucially, the stem *dush* 'to be' is just a verb.

- Moreover, it is embedded in exactly the same kind of relative clause as dependent numerals.

- Thus, just like in the dependent numerals, it's the event argument of this embedded verb that *-xper-* marks as dependent!

- The head of the relative clause—the subject of the main clause—must be interpreted distributively to satisfy the dependency requirement of the *-xper-*-marked verb in its relative clause complement.

But, if main clause subject is interpreted distributively to satisfy a requirement of a dependent-marked embedded clause, it will also be interpreted distributively for the main clause.

- Voilà, prima facie distributive key-marking without distributive key-marking.

We assume the following structure for *xper*-marked coordinated nominals in (52) (in Piipaash, nominative case marks (nominal) predicates)

(53)   [$_{NP}$ pro$_i$ [$_{CP}$ John-sh Bill-sh$_i$ nyi-dush-**xper**-k]]
       [pro [John-NOM Bill-NOM PL.OBJ-be.DU-each-ss]]
       lit. 'Them being John, Bill'

Note that we assume the coordination is not contributed by the *dush* verb.

- coordination, both conjunction and disjunction, is more generally marked by juxtaposition in Piipaash. We have already seen examples of this—e.g., (42).

- Instead, we take the contribution of *dush* to support the equative interpretation.

Once marked pluractional (and after event closure and application of its external argument), we have the following denotation for *Johnsh Billsh nyidushxperk* 'being John, Bill'.

(54) *Johnsh Billsh nyidushxperk* $\rightsquigarrow \lambda x \exists e[\text{BE}(e) \wedge \text{TH}(e) = x \wedge x \leq \mathbf{j} \oplus \mathbf{b} \wedge \overline{e >_{\text{TH}} 1}]$
      'True of individuals that are less than or equal to John and Bill that participate in at least two events of being that have different themes.'

Crucially, the only way this can be satisfied is if it is interpreted in the scope of a distributive operator (and if we pass at least two individuals to x).

- Both constraints can simultaneously be satisfied if the head of the relative clause in which *Johnsh Billsh nyidushxperk* is embedded is interpreted distributively.

- This is precisely the observed interpretation of (52).

(55) $\forall x[x \leq \text{i} \wedge \mathbf{one}(x) \rightarrow$
      $\exists e[\text{BE}(e) \wedge \text{TH}(e) = x \wedge x \leq \mathbf{j} \oplus \mathbf{b} \wedge \overline{e >_{\text{TH}} 1} \wedge$
      $\exists z \exists e'[sticks(z) \wedge \text{TH}(e') = z \wedge |\text{TH}(e')| = 3 \wedge$
      $\exists e''[carry(e'') \wedge ag(e'') = x \wedge th(e'') = z]]]]$
      'True if for each individual x in i, there is (i) an event of x being and x is John or Bill, (ii) a second event involving three stick participants z, and (iii) a third event of carrying in which x carries z.'

Note that here that the *xper*-marked verb does very little truth conditional work. It merely forces the subject to be interpreted distributively.

- But, this is exactly what we wanted. We want to understand why the nominal that is the distributive key contains a *xper*-marked verb, when in other cases it was the distributive key.

- Crucially, our account in terms dependent pluractionality allows us to get the correct truth conditions while maintaining a uniform denotation for *-xper-*.

4   Against a syntactic split-scope account

We have repeatedly seen that dependent expressions, both indefinites and pluractionals, involve interpreting part of an expression's content inside the scope of a distributivity operator—the existential operator—and part outside of it—the evaluation plurality constraint.

- We use post-suppositions to generate this effect.

- One might wonder whether there are other mechanism, for instance, from the split-scope literature that could also be used to analyze dependent indefinites / pluractionals.

Kuhn 2017 argues for a split-scope account of dependent indefinites in the copy-movement style of Abels and Martí 2010.

- That is, the dependent indefinite moves outside of the scope of the distributivity operator, leaving a copy below, but at spell-out we interpret only part of the copy outside the scope of the distributivity operator, namely that part that forces co-variation (i.e. the plurality condition).

First, it is important to note that he *requires* a split-scope account. Otherwise there are basic data he cannot account for.

- his analysis is based on special conditions called **inside** and **outside**—essentially,

  - **outside**$(y/x) > 1$ tests that y is dependent on x, in the formal sense.
  - **inside**$(y/x) = n$ is a test that n distinct y's stored across the set of variable assignments that agree on x.

We thus get an account of dependent indefinites like "each student saw two two zebras" like the following:

(88)
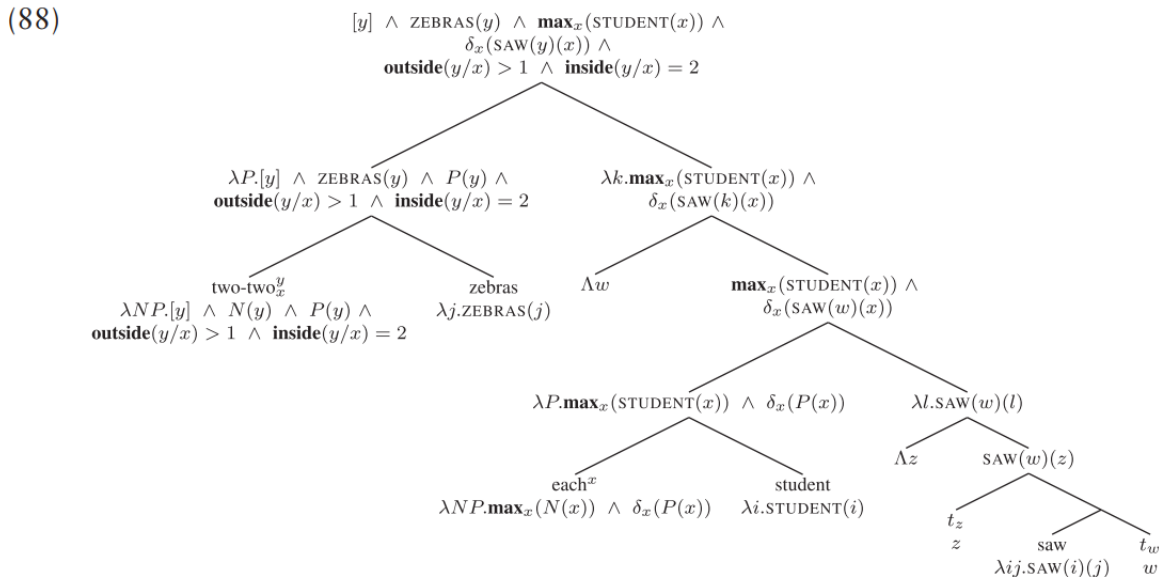
This kind of account is very much like mine in that the critical conditions (here inside and outside) need to be interpreted outside the scope of a distributivity operator.

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- They must take widest scope, which is derived in Kuhn's account by having the dependent indefinite undergo QR.

- Ok, but what about when the dependent indefinite cannot undergo QR?

Consider the following examples from Hungarian.

(56)   Minden rendezö benevezte két-két   filmjét.
         every   director entered   two-two his   films
         'Every director entered two of his films.'

(57)   A diátok   két elöételt   és egy-egy föélt   rendeltek.
         the students   two appetizers and one-one main-dish ordered
         'The students orderd two appetizers and one main dish.'

These require split scope for Kuhn, but my account gets them immediately. He has to have copy-movement with selective deletion.

(111)
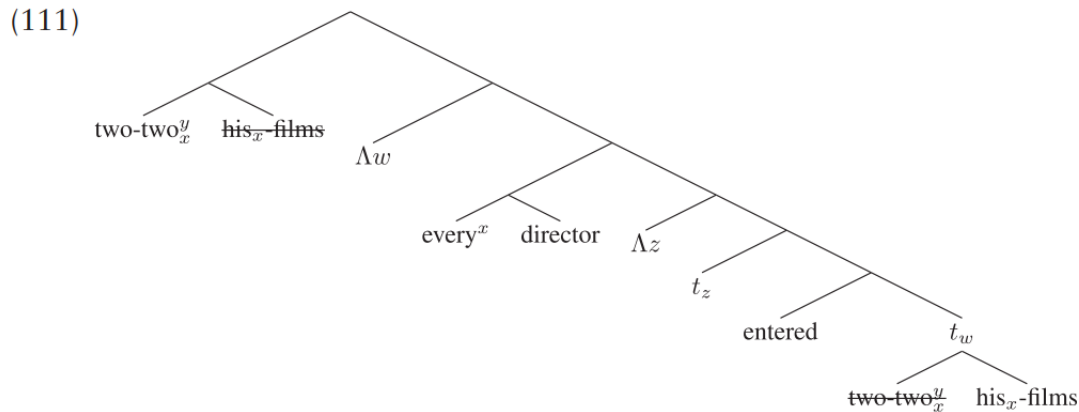
There is only one positive argument for this kind of account and against the post-suppositional account. In particular, Hungarian dependent indefinites are not licensed in islands.

(58)   Minden professzor két-két diákról   mondta, hogy meglepné ha diplomát
         every   professor two-two students-of said   that surprised if diploma
         szereznének
         receive
         'Every professor said of two students that he would be surprised if they graduated.'

(59)   *Minden professzor azt mondta, hogy meglepné, ha két-két diák   diplomát szerezne
         every   professor DEM said   that surprised if two-two student diploma receive
         'Every professor said of two students that he would be surprised if they graduated.'

- This would make sense if the dependent indefinite were undergoing QR, as in a split-scope account.

- In contrast, a post-suppositional account would have to say that, in Hungarian, post-suppositions are discharged at island boundaries—clearly ad hoc (though maybe not so bad...phases or whatnot, right?).

The Piipaash data we have considered here, as well the Seri facts described in Pasquereau 2019, 2021 can bear on this argument.

- First, as we have repeatedly emphasized, the dependence is marked on verbs in these languages. While a QR account is prima facie plausible for languages like Kaqchikel or Hungarian, where dependence is marked on indefinites and numerals, it is harder to argue that verbs undergo QR in languages like Piipaash.

- Second, even granting that verbs can move in Piipaash, we run into problems with a split scope account. In particular, dependent numerals are deeply embedded in relative clauses in Piipaash, which are islands to movement.

    - This fact weighs in favor of a postsuppositional account, like that developed here, which is not inherently constrained by islands.

The fact that we seem to have island sensitive and non-island-sensitive dependent expressions is itself an important empirical conclusion of this work. Figuring out the sources of these differences is an important question for future research.

- Perhaps some language use a QR mechanism, and other languages use post-suppositions, a purely semantic mechanism.

- Or, perhaps post-suppositions are discharged at different syntactic constituents in different languages.

I am not so sure how to distinguish these various alternatives, but it is intriguing.

5 Conclusions

The morpheme *-xper-* in Piipaash provides good evidence for a novel kind of pluractionality we call *dependent pluractionality*.

- Given that dependent indefinites are familiar from the literature, and predicates of events, like verbs, in virtue of undergoing existential closure, have a kind of indefinite flavor, perhaps this is expected!

- Once we make this move, we can solve two puzzles about *-xper-*:

    - it has a wide distribution—that's because lots of expressions can be predicates of events
    - it can, in cases look like it's marking distributive shares or keys—i.e., Gil's Puzzle—our solution is that it always marks shares, but in virtue of marking event predicates can appear inside a key that itself embeds a verbal predicate.

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